Autonomous Navigation of Teams of Unmanned Aerial or Underwater Vehicles for Exploration of Unknown Static & Dynamic Environments

A. Ch. Kapoutsis, S. A. Chatzichristofis, L. Doitsidis, J. Borges de Sousa and E. B. Kosmatopoulos

Abstract— **In this paper, we present a new approach that is able to efficiently and fully-autonomously navigate a team of Unmanned Aerial or Underwater Vehicles (UAUV's) when deployed in exploration of unknown static and dynamic environments towards providing accurate static/dynamic maps of the environment. Additionally to achieving to efficiently and fully-autonomously navigate the UAUV team, the proposed approach possesses certain advantages such as its extremely computational simplicity and scalability, and the fact that it can very straightforwardly embed and type of physical or other constraints and limitations (e.g., obstacle avoidance, nonlinear sensor noise models, localization fading environments, etc).**

I. INTRODUCTION

Typically, when a single Unmanned Aerial or Underwater Vehicle (UAUV) or a team of UAUVs is deployed to map (explore) an unknown static or dynamic environment, the static (landmarks) and/or dynamic (targets) features of the environment as well as the positions of the UAUVs themselves are estimated through a so-called Simultaneous Localization And Mapping and Target Tracking (SLAM-TT) algorithm, which employs an EKF or similar approach to simultaneously estimate all the above-mentioned quantities, see e.g. [1], [2], [3], [4] and the references there in. Over the past years, very powerful approaches have been developed that can quite efficiently provide the estimates of the landmarks', targets' and UAUVs' positions, provided that the *trajectories of the UAUVs are efficiently designed*. However, efficient design of the UAUV trajectories is not trivial: in most cases an *off-line* design of the UAUV trajectories is performed. Off-line design of the UAUV trajectories is, of course, by no means a guarantee of performance as the UAUVs may enter into highly unobservable states, they may spend "too much time" in areas with no important information for the exploration task, while they may pass very fast through very crucial areas for the exploration task, producing thus a very poor map of these areas, etc.

For this reason, the last few years special attention have been paid in developing techniques for *active exploration* (active SLAM-TT), see e.g., [4], [5], [6], [7] and the references therein: using the information received so far, the UAUV next positions are designed so they optimize the mapping information of the SLAM-TT algorithm. One possible way to attack such a problem is as follows: check all feasible next UAUV positions (e.g., all next UAUV positions that do not violate obstacle avoidance, maximum speed, communication, etc constraints) and find the ones that optimize some information metric that corresponds to the accuracy of the SLAM-TT algorithm; then, move to the positions that optimize this information metric, and so on. Different types of such information metrics have been proposed, with the most popular being the trace of the EKF error covariance matrix, see e.g., [4], [5]. In such a case the UAUVs are moving to the next positions that minimize the average (expected) EKF estimation error.

There two big issues with the above mentioned approach: the first is scalability, since it is computationally not feasible to check all possible combinations of next UAUVs positions (this is practically infeasible even in the single UAUV case). There are, of course, many different approaches that relax the computational requirement of checking all possible next positions at the expense of sacrificing efficiency. However, even in the unrealistic case where infinite computing power would be available, as these algorithms are based on EKF – which, in turn, is based on linearizing the nonlinear multi-UAUV/sensor dynamics – the presence of nonlinear constraints (e.g., for obstacle avoidance or for not leaving a pre-specified area) may be destructive to the efficiency of the overall active exploration mission. The results of such a case are depicted in Figure 1: three UAUVs have been deployed for estimating the location of 30 static landmarks and their trajectories are designed so they minimize the trace of the EKF error covariance matrix, while they avoid obstacles (landmarks) and they remain within the cube $[-1,+1]^3$. Although, in the time-interval $[0,79]$ the overall algorithm behaves quite efficiently, it starts diverging as soon as the UAUVs "hit" the boundaries of the area they have to remain within.

Another class of methodologies for active exploration are based on optimal control or dynamic programming principles, see e.g. [5], [6], [7] and the references there in: the UAUV trajectories are on-line calculated so that they optimize the accuracy of the static/dynamic map of the external environment during the overall mission while they do not violate the physical and other constraints imposed by the particular application. As in the previously mentioned approaches, the "curse of dimensionality" problem

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(practically infeasible to compute in real-time the optimal UAUVs trajectories) is present there too: calculating the optimal UAUV trajectories is an NP-complete problem and, for this reason, relaxations or approximations of the optimal trajectory computations are required. Despite the quite successful and promising results of such approaches in *small-scale* applications or applications where sufficient *a priori* information is provided (e.g., for cases where a single autonomous vehicle is used or an initial "good" estimate of the environmental map is provided, etc), their extension to large-scale real-life applications is still an open issue.

In this paper, we present a new approach that overcomes the shortcomings of the existing methodologies. There are two basic attributes that render the proposed approach attractive: its extremely computational simplicity and the fact that it can straightforwardly incorporate arbitrary constraints and limitations that are met in real-life applications. The proposed approach is based on the so-called Cognitive-based Adaptive Optimization (CAO) algorithm that has been successfully implemented in multi-UAUV optimal surveillance coverage problems [8], [9]. The approach of [8], [9] was shown to efficiently handle arbitrary number of UAUVs and maps of arbitrary complexity and number of features. As the proposed in this paper approach is based on the extension of the approach of [8], [9], it inherits all these nice properties.

II. Autonomous Multi-UAUV Navigation for Exploration of Unknown Environments

Consider a team of N_R UAUVs moving in a 3D environment in order to estimate as accurately as possible the 3D positions of N_L static features (landmarks) as well as the 3D positions of N_T moving features (dynamic targets). The UAUV sensors are equipped proprioceptive measurements (e.g., from GPS or inertial sensors) to propagate their state (position and orientation) estimates, and are equipped with exteroceptive sensors (e.g., laser range finders, cameras, sonars, etc) that enable them to measure their distance or bearing from other robots and landmarks. To simplify our analysis, we will assume that the position and orientation (pose) of each UAUV are known with high accuracy within the global frame of[1] reference (e.g., from GPS and IMU measurements).

Let x_i^L and x_i^T denote the 3D position of the ith landmark (resp. target), x_i^R denote the position or pose of the ith UAUV and

$$\mathbf{X} = [x_1^L, \ldots, x_{N_L}^L, x_1^T, \ldots, x_{N_T}^T], \quad \mathbf{X}^R = [x_1^R, \ldots, x_{N_R}^R]$$

denote the matrices of all features to be estimated and all UAUVs positions (poses), respectively. Furthermore, let $\mathbf{Y}$ denote the vector of all UAUVs' sensor measurements. In the most general case we have that the sensor measurements are related to the matrices $\mathbf{X}$ and $\mathbf{X}^R$ through a nonlinear function that admits the form

$$\mathbf{Y} = H(\mathbf{X}, \mathbf{X}^R, \Xi)$$

where H is the nonlinear vector sensor function and Ξ is the sensor measurement noise vector. Let also $\hat{\mathbf{X}}$ denote the estimate of $\mathbf{X}$ as generated by a standard SLAM-TT algorithm (e.g., an EKF-based one). Apparently, different UAUV trajectories $\mathbf{X}^R(t)$ result in different accuracy for the SLAM-TT algorithm. The *active exploration* problem is that of generating on-line the trajectories of the UAUVs $\mathbf{X}^R(t)$ so that the estimation accuracy of the SLAM-TT algorithm is optimized. Additionally to optimizing SLAM-TT accuracy, the design for exploration using UAUVs will have to take into account the – sometimes very strict – limitations of the environment the UAUVs operate on: safe navigation, nonlinear sensor noise characteristics, and limited visibility of the UAUVs sensors are some of the limitations that render multi-UAUV autonomous navigation for exploration a very challenging task. Below, we list all different major limitations/challenges that any strategy for such a problem has to take into account:

(NL-Noise) The typical assumption made in most robotic applications that the sensor noise is additive Gaussian noise is very restrictive and not realistic in many UAUV applications. For instance, in UAUVs sonar- and vision-based sensors, the sensor noise affect the sensor measurements in a *NonLinear* fashion: typically, the noise affecting such sensors is proportional to the sensor-to-sensing point distance, i.e., *the larger is the UAUV-to-sensing point distance, the larger is the sensor noise*. As a result, it is more realistic to assume a multiplicative sensor noise model that takes the form

$$y = h(x,q) + h_\xi(x,q)\xi \tag{1}$$

where y is the sensor measurement, x, q are the positions of the UAUV and the sensing point (landmark/target/another UAUV), respectively, $h(x,q)$ is the sensor model in the noise-free case, $h_x i(x,q)$ is a nonlinear function of x and q [e.g., $h_\xi(x,q)$ is the distance between x and q] and ξ is a standard Gaussian noise.

(LimVis) In addition to the (NL-Noise) limitation, the UAUV exteroceptive sensors are of limited visibility. As a result, additionally to the nonlinear sensor noise assumption (1), the sensor model for the exteroceptive sensors should be augmented to count for the limited visibility constraint. Moreover, the sensor model must be augmented to count for the case where there is no line-of-sight between the UAUV and the sensing point (e.g., there is an obstacle in between). As a result, the actual sensor model becomes:

$$y_{x-q} = \begin{cases} \text{undefined} & \text{if } \|x-q\| \geq \mathit{thres} \\ \text{undefined} & \text{if there is no line-of-} \\ & \text{sight between } x \text{ and } q \\ h(x,q) + h_\xi(x,q)\xi & \text{otherwise} \end{cases} \tag{2}$$

where y_{x-q} denotes the sensor measurement from an UAUV at position x to a sensing point at position q, *thres* denotes the

[1]In other words, we will assume that the UAUVs are perfectly localized and thus we will deal only with the problem of Simultaneous Mapping and Target Tracking – (SM-TT). It has to be emphasized that the proposed approach can be easily extended to deal with the SLAM-TT case, i.e., the case where simultaneously to estimating the static/dynamic map, the overall approach estimates the UAUVs poses by combining the proprioceptive and exteroceptive measurements of the UAUVs.

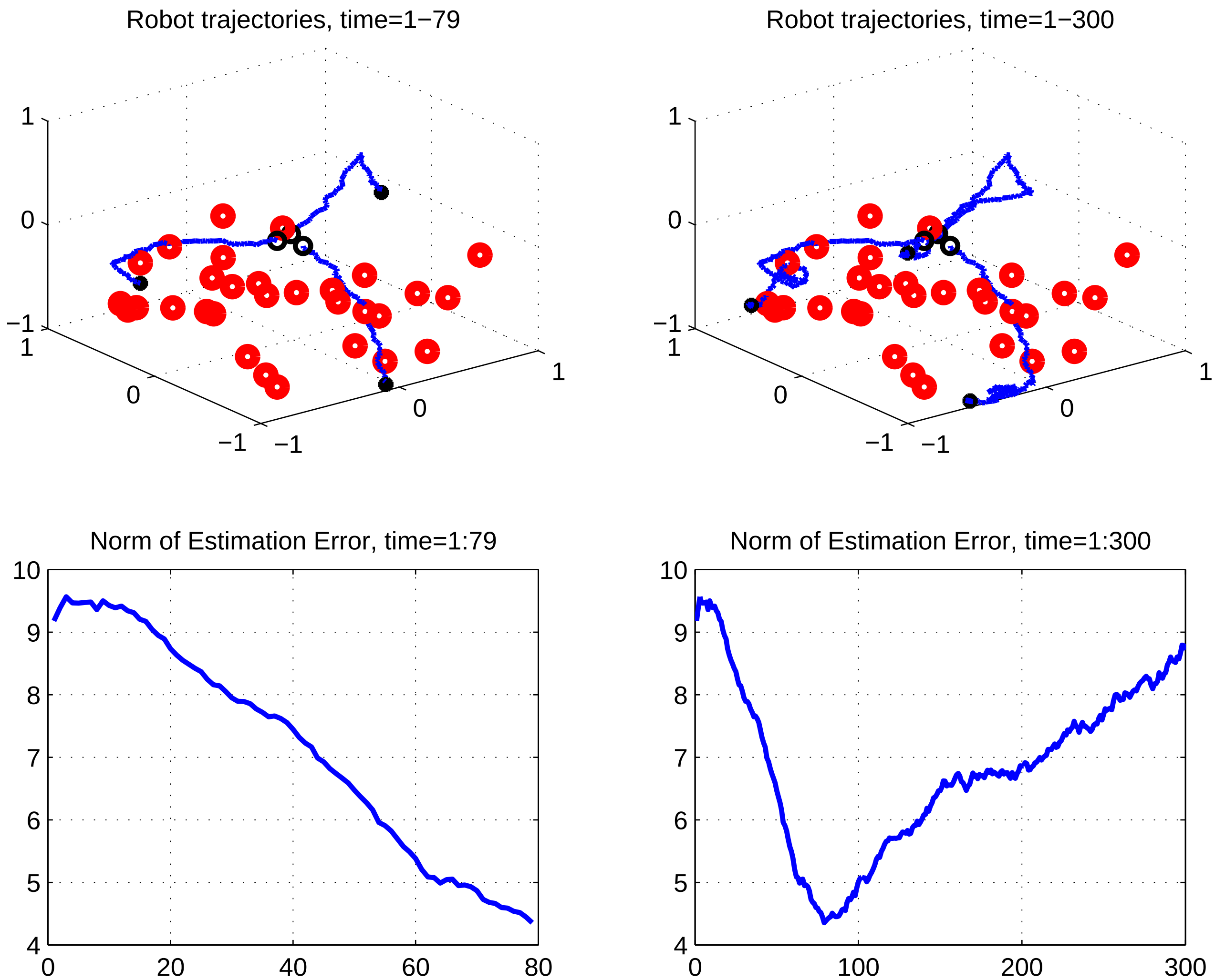

Fig. 1. Autonomous exploration by moving towards minimizing the trace of EKF error covariance matrix: $N_R = 3, N_L = 30, N_T = 0$, by assuming unlimited visibility, perfect localization and infinite computing power. The estimation error starts diverging as soon as the UAUVs hit the boundary of the cube $[-1,+1]^3$ the UAUVs are constrained to remain within.

visibility threshold beyond which the vision or sonar sensor does not "see" and $\|\cdot\|$ denotes the Euclidean norm.

(ObsAvoid) As in any real-life robot application, the UAUV navigation system must make sure that the UAUVs avoid obstacles as well as they remain within a pre-specified operational area. Usually, it is realistic to assume that the UAUVs can detect with accuracy the position of the obstacles nearby.

(Scalable) Finally, a main issue for any multi-UAUV navigation algorithm for exploration is scalability. Of course, scalability is an issue in any multi-robot application. In the case of underwater multi-UAUV applications, the scalability issue becomes way more significant mainly due to the limited bandwidth of UAUVs communication systems that allow only a few hundreds of bits/second to be transmitted/received.

Having all these limitations in mind, we now proceed to present the proposed methodology.

III. Problem Definition

In order to describe the proposed approach we need some preliminaries. Let $\mathscr{P} = \{x^{(i)}\}_{i=1}^{N_R}$ denote the configuration[2] of the UAUV team, where $x^{(i)}$ denotes the position of the i-th UAUV. We will say that a landmark or a target $q = (x,y,z)$ is visible if there exists at least one UAUV so that

- the UAUV and the point q are connected by a line-of-sight;
- the UAUV and the point q are at a distance smaller than a given threshold value (defined as the maximum distance the UAUVs' sensor can "see").

Given a particular team configuration $\mathscr{P}$, we let $\mathscr{V}$ denote the subset of all *visible landmarks and targets*, i.e., $\mathscr{V}$ consists of all landmarks and targets q that are visible from the UAUVs.

[2]For simplicity, we will assume that all UAUVs are of fixed and known orientation. All the results can be easily extended to the case of variable orientation.

Also, for any landmark or target $q = (x, y, z)$, let $\hat{q}$ denote its estimate as produced by SLAM-TT. We will say that the landmark or the target q is currently accurately-estimated, if the normed-error $\|q - \hat{q}\||$ is below a certain accuracy threshold. We will denote with $\mathscr{A}$ the set of all landmarks and targets that are currently accurately-estimated. Please note that in case a landmark becomes accurately-estimated then it wil remain accurately-estimated thereafter (i.e., it remains within $\mathscr{A}$ thereafter); however, this is not true for a moving target which may belong to $\mathscr{A}$ at some point and then leave this subset later.

By using the above definitions, we introduce the following[3] *active exploration cost criterion*:

$$\begin{aligned} J(\mathscr{P}) &= \int_{q \in \mathscr{V}, q \notin \mathscr{A}} \min_{i \in \{1,\ldots,N_R\}} \|x^{(i)} - q\|^2 dq \\ &\quad + K \int_{q \notin \mathscr{V} \cup \mathscr{A}} dq \end{aligned} \tag{3}$$

where K is a user-defined positive constant. Having the UAUV team minimizing the above criterion, is equivalent to have the UAUVs come as close as possible to those landmarks/targets that are currently visible and have not been accurately-estimated [first term in the RHS of (3)] and, concurrently moving the UAUVs so that they "see" those landmarks/targets that are currently not visible and not accurately-estimated [second term in the RHS of (3)]. In other words, the first term is responsible for moving the UAUVs closer to the landmarks/targets so that they reduce the sensor noise effect and they can "see them better", while the second term is responsible for moving the UAUVs closer to landmarks/targets that "have not seen before" (or "have been poorly seen"). The constant K serves as a weight for giving less or more priority to one of the terms of the RHS of (3). Please note that if the UAUVs' trajectories achieve to render the value of J zero (or sufficiently small), then the overall active exploration mission has been successfully accomplished provided that the position of all UAUVs is accurately known.

IV. The Cognitive-based Adaptive Optimization Approach

Having defined the active exploration criterion, we will now proceed on presenting the proposed algorithm for autonomously navigating the UAUVs towards minimizing such a criterion. The algorithm to be used is based on the so called Cognitive-based Adaptive Optimization (CAO) approach originated in the references [10], [11], [12], The version of the CAO algorithm used within the proposed approach takes the same form as the one of [8], [9] and is a an extension of the original CAO version of presented and analyzed in [11], [12]. The main difference is that the work of [8], [9] extended the CAO approach of [11], [12] so that it efficiently takes care of the various constraints of the type (ObsAvoid). Below, we provide the main details of the CAO algorithm as employed in the framework of the active exploration problem.

We start by noticing that the active exploration criterion (3) is a function of the UAUVs positions, i.e.,

$$J_k = \mathscr{J}\left(x_k^{(1)}, \ldots, x_k^{(N_R)}\right) \tag{4}$$

where $k = 0, 1, 2, \ldots$ denotes the time-index, J_k denotes the value of the active exploration criterion at the k-th time-step, $x_k^{(1)}, \ldots, x_k^{(N_R)}$ denote the position vectors of the UAUVs $1, \ldots, N_R$, respectively, and $\mathscr{J}$ is a nonlinear function which depends – apart from the UAUVs positions – on the particular environment where the UAUVs live (e.g., position of landmarks/targets).

Due to the dependence of the function $\mathscr{J}$ on the particular environment characteristics, the *explicit form of the function $\mathscr{J}$ is not known* in practical situations; as a result, standard optimization algorithms (e.g., steepest descent) are not applicable to the problem in hand. However, in most practical cases, like the one treated in this paper, the current value of the active exploration criterion can be estimated from the UAUVs sensor measurements. In other words, at each time-step k, an estimate of J_k is available through UAUVs sensor measurements,

$$J_k^n = \mathscr{J}\left(x_k^{(1)}, \ldots, x_k^{(N_R)}\right) + \xi_k \tag{5}$$

where J_k^n denotes the estimate of J_k and ξ_k denotes the noise introduced in the estimation of J_k due to the presence of noise in the UAUVs sensors. Please note that, although it is natural to assume that the noise sequence ξ_k is a stochastic *zero-mean* signal, it is not realistic to assume that it satisfies the typical Additive White Noise Gaussian (AWNG) property even if the UAUVs sensor noise is AWNG: as $\mathscr{J}$ is a nonlinear function of the UAUVs positions (and thus of the UAUVs sensor measurements), the AWNG property is typically lost.

Apart from the problem of dealing with a criterion for which an explicit form is not known but only its noisy measurements are available at each time, efficient UAUV navigation algorithms have additionally to deal with the problem of restricting the UAUVs positions so that obstacle avoidance constraints are met. In other words, at each time-instant k, the vectors $x_k^{(i)}, i = 1, \ldots, N_R$ should satisfy a set of constraints which, in general, can be represented as follows:

$$\mathscr{C}\left(x_k^{(1)}, \ldots, x_k^{(N_R)}\right) \leq 0 \tag{6}$$

where $\mathscr{C}$ is a set of nonlinear functions of the UAUVs positions. As in the case of $\mathscr{J}$, the function $\mathscr{C}$ depends on the particular environment characteristics (e.g., location of obstacles, terrain morphology) and an explicit form of this function may be not known in many practical situations; however, it is natural to assume that the active exploration

[3]Please note that the subset $\mathscr{A}$ cannot be calculated in real-life as its calculation requires knowledge of the true landmark/target positions. However, in practice the subset $\mathscr{A}$ can be estimated with high accuracy from e.g., the EKF error covariance matrix (e.g., if all three elements of the diagonal of the EKF error covariance matrix that correspond to a particular landmark/target are below a certain accuracy threshold, then this landmark/target belongs to $\mathscr{A}$). Similarly the term $\int_{q \notin \mathscr{V} \cup \mathscr{A}} dq$ cannot be computed in practice as this term involves those landmarks/targets that are invisible. This problem can be overcome by noticing that $\int_{q \notin \mathscr{V} \cup \mathscr{A}} dq = \int_q dq - \int_{q \in \mathscr{V} \cup \mathscr{A}} dq$ and the integral $\int_q dq$ is constant.

algorithm is provided with information whether a particular selection of UAUVs positions satisfies or violates the set of constraints (6).

Given the mathematical description presented above, the active exploration problem can be mathematically described as the problem of moving $x_k^{(1)},\ldots,x_k^{(N_R)}$ to a set of positions that solves the following constrained optimization problem:

$$\begin{array}{ll}\text{minimize} & (4)\\ \text{subject to} & (6)\,.\end{array} \tag{7}$$

As already noticed, the difficulty in solving, in real-time and in real-life situations, the constrained optimization problem (7) lies in the fact that explicit forms for the functions $\mathscr{J}$ and $\mathscr{C}$ are not available. To circumvent this difficulty, the CAO approach, appropriately modified to be applicable to the problem in hand, is adopted. Indeed this algorithm is capable of efficiently dealing with optimization problems for which the explicit forms of the objective function and constraints are not known, but noisy measurements/estimates of these functions are available at each time-step. In the following, we describe the CAO approach as applied to the multi-robot coverage problem described above.

As a first step, the CAO approach makes use of function approximators for the estimation of the unknown objective function $\mathscr{J}$ at each time-instant k according to

$$\hat{J}_k\left(x_k^{(1)},\ldots,x_k^{(N_R)}\right)=\vartheta_k^{\tau}\phi\left(x_k^{(1)},\ldots,x_k^{(N_R)}\right). \tag{8}$$

Here $\hat{J}_k\left(x_k^{(1)},\ldots,x_k^{(N_R)}\right)$ denotes the approximation/ estimation of $\mathscr{J}$ generated at the k-th time-step, ϕ denotes the nonlinear vector of L *regressor terms*, ϑ_k denotes the vector of *parameter estimates* calculated at the k-th time-instant and L is a positive user-defined integer denoting the *size* of the function approximator (8). The vector ϕ of regressor terms must be chosen so that it is a universal approximator, such as polynomial approximators, radial basis functions, kernel-based approximators, etc.

The parameter estimation vector ϑ_k is calculated according to

$$\vartheta_k=\underset{\vartheta}{\operatorname{argmin}}\frac{1}{2}\sum_{\ell=\ell_k}^{k-1}\left(J_\ell^n-\vartheta^{\tau}\phi\left(x_\ell^{(1)},\ldots,x_\ell^{(N_R)}\right)\right)^2 \tag{9}$$

where $\ell_k=\max\{0,k-L-T_h\}$ with T_h being a user-defined nonnegative integer. Standard least-squares optimization algorithms can be used for the solution of (9).

As soon as the estimator $\hat{J}_k$ is constructed according to (8), (9), the set of new UAUVs positions is selected as follows: firstly, a set of N candidate UAUVs positions is constructed according to[4]

$$x_k^{i,j}=x_k^{(i)}+\alpha_k\zeta_k^{i,j}, i\in\{1,\ldots,N_R\}, j\in\{1,\ldots,N\}, \tag{10}$$

[4]According to [11], [12] it suffices to choose N to be any positive integer larger or equal to 2×[the number of variables being optimized by CAO]. In our case the variables optimized are the robot positions $x_k^{(1)},\ldots,x_k^{(N_R)}$ and thus it suffices for N to satisfy $N\geq 2N_R\times\dim\left(x_k^{(i)}\right)$.

where $\zeta_k^{i,j}$ is a zero-mean, unity-variance random vector with dimension equal to the dimension of $x_k^{(i)}$ and α_k is a positive real sequence which satisfies the conditions:

$$\lim_{k\to\infty}\alpha_k=0,\quad \sum_{k=1}^{\infty}\alpha_k=\infty,\quad \sum_{k=1}^{\infty}\alpha_k^2<\infty. \tag{11}$$

Among all N candidate new positions $x_k^{1,j},\ldots,x_k^{N_R,j}$, the ones that correspond to non-feasible positions – i.e., the ones that violate the constraints (6) – *are neglected* and then the new UAUVs positions are calculated as follows:

$$\left[x_{k+1}^{(1)},\ldots,x_{k+1}^{(N_R)}\right]=\underset{\substack{j\in\{1,\ldots,N\}\\ x_k^{i,j}\text{not neglected}}}{\operatorname{argmin}}\hat{J}_k\left(x_k^{1,j},\ldots,x_k^{N_R,j}\right)$$

The idea behind the above logic is simple: at each time-instant a set of many candidate new UAUVs positions is generated. The candidate, among all feasible ones, that provides the best estimated value $\hat{J}_k$ of the coverage criterion is selected as the new set of UAUVs positions. The random choice for the candidates is essential and crucial for the efficiency of the algorithm, as such a choice guarantees that $\hat{J}_k$ is a reliable and accurate estimate for the unknown function $\mathscr{J}$; see [11], [12] for more details. On the other hand, the choice of a slowly decaying sequence α_k, a typical choice of adaptive gains in stochastic optimization algorithms is essential for filtering out the effects of the noise term ξ_k [cf. (5)]. The next theorem summarizes the properties of the CAO algorithm described above; the proof can be found in [9].

Theorem 1: Let $x^{(1^*)},\ldots,x^{(N_R^*)}$ denote any – local – minimum of the constrained optimization problem (7). Let $N\geq 2N_R\times\dim\left(x_k^{(i)}\right)$ and, moreover, the vector ϕ satisfy the Universal Approximation Property. Assume also that the functions $\mathscr{J},\mathscr{C}$ are either continuous or discontinuous with a finite number of discontinuities. Then, the CAO-based multi-UAUV exploration algorithm as described above guarantees that the UAUVs positions $x_k^{(1)},\ldots,x_k^{(N_R)}$ will converge to one of the local minima $x^{(1^*)},\ldots,x^{(N_R^*)}$ almost surely, provided that the size L of the regressor vector ϕ is larger than a lower bound $\bar{L}$.

V. Simulation Experiments

In order to test the efficiency of the proposed approach, a realistic scenario was considered by using a map of a real area. The area (located in Zrich, Switzerland) was mapped using a state-of-the-art visual-SLAM algorithm which tracks the pose of the camera while, simultaneously and autonomously, building an incremental map of the surrounding environment. More details regarding the extraction methodology are given in [13],[14]. The details of the simulation environment are as follows:

- The number of UAUVs is equal to $N_R=3$ while the number of targets is equal to $N_T=2$ The target trajectories are generated using a zero-acceleration model [4].

- For the number of landmarks 2 different scenarios were tried. First, we consider that every point of the map is a landmark. In this case, the map includes $N_L = 7542$ landmarks. In the second scenario, we consider that the map consists only of $N_L = 1000$ landmarks.
- The main constraints imposed to the UAUVs are that they remain within the terrain's limits, i.e., within $[X_{min}, X_{max}]$ and $[Y_{min}, Y_{max}]$ in the x- and y-axes, respectively. At the same time, UAUVs remain within $[z + d, z_{Max}]$ along the z-axis, in order to avoid hitting the terrain. The value of d was equal to 0.3.
- The UAUV-to-landmark and UAUV-to-target sensors were assumed to be range sensors concatenated by multiplicative noise as follows:

$$y_{x-q} = \begin{cases} \text{undefined} & \text{if } \|x-q\| \geq thres \\ \text{undefined} & \text{if there is no line-of-sight between } x \text{ and } q \\ d(x,q) + d(x,q)\xi & \text{otherwise} \end{cases} \tag{12}$$

  where ξ is a Gaussian noise of variance 0.01. The visibility thresholds were set equal to 0.4 for the UAUV-to-landmark sensors and infinite for the UAUV-to-target sensors. Also, a line-of-site between the UAUV and a landmark/target/another UAUV was assumed in case there is no landmark/target or another UAUV in a distance less than 0.1 from the line connecting the UAUV with the landmark/target/another UAUV.
- All UAUVs were assumed to have constant orientation which, moreover, does not have any effect on the sensing capabilities or the sensor model (12). Moreover and for simplicity a simple linear model for the UAUV dynamics was assumed, and no effect from external disturbances (e.g., currents or turbulences) was considered.

Figures 2 and 3 exhibit some 2D and 3D, respectively, snapshots of a particular simulation experiment.

In the case of 2D representation, the UAUV are illustrated as gray circles while targets are depicted as yellow squares. In the initial state, Time $= 0$, the map is depicted with red color. This implies that no landmark have been estimated. During a next time step, Time $= 62$, some red spots disappear, and the map shows up, as height map. Additionally, one can distinguish some points in green color, describing the map positions that currently visible but are non-accurately estimated. When Time $= 96$ only 2432/7542 landmarks are not estimated and finally, when Time $= 500$ only 3.09% of the map is still unknown.

Similarly, in 3D representation, the black landmarks correspond to the completely unknown ones, the red one describe the visible but non-accurately estimated landmarks while the green ones correspond to the ones that are estimated.

The proposed algorithm is compared against the case of a purely random algorithm (i.e., the next UAUV positions are randomly chosen by making sure that the trajectories do not violate any of the obstacle avoidance, maximum speed, etc, constraints). 10 different sets of simulation experiments for each of the scenarios that we examine ($N_L = 7542$ and $N_L = 1000$) were executed and Figures 4 and 5 demonstrate the results. Four different evaluation criteria are used: (sum of) ERROR: the integral of the norm of estimation error during the overall mission (i.e., from $t = 0$ to $t = 500$); (sum of) Cost Function: the integral of $J(t)$ during the overall mission (i.e., from $t = 0$ to $t = 500$) ; Cost Function Final value: the value of the cost function J at $t = 500$; Remaining Items: the total number of landmarks that are not accurately-estimated at $t = 500$.

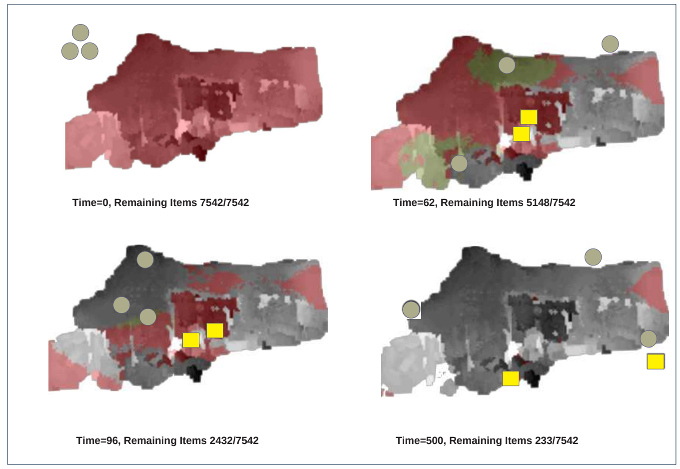

Fig. 2. 2D representation of the autonomous exploration using CAO: $N_R = 3$, $N_L = 7542$, $N_T = 2$.

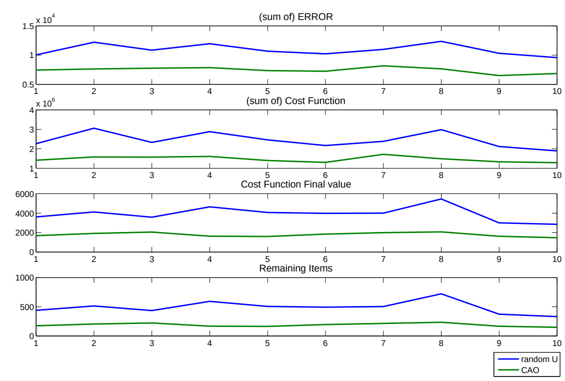

Fig. 4. Autonomous exploration using CAO and Random trajectories: $N_R = 3$, $N_L = 7542$, $N_T = 2$. Comparison of different evaluation criteria for 10 different experiments.

The experimental results clearly exhibits the superiority of the proposed approach. The CAO-based multi-UAUV exploration achieves to estimate accurately most of the landmarks. Conclusions are reinforced by observing Figure 6 which describes the norm of estimation error for the 2 methods we are comparing in the case that $N_L = 7542$.

Concluding, it is very important to highlight that as we increase the number of landmarks, the performance of the purely random algorithm decrease. At the same time, the performance of the proposed method tends to remain stable.

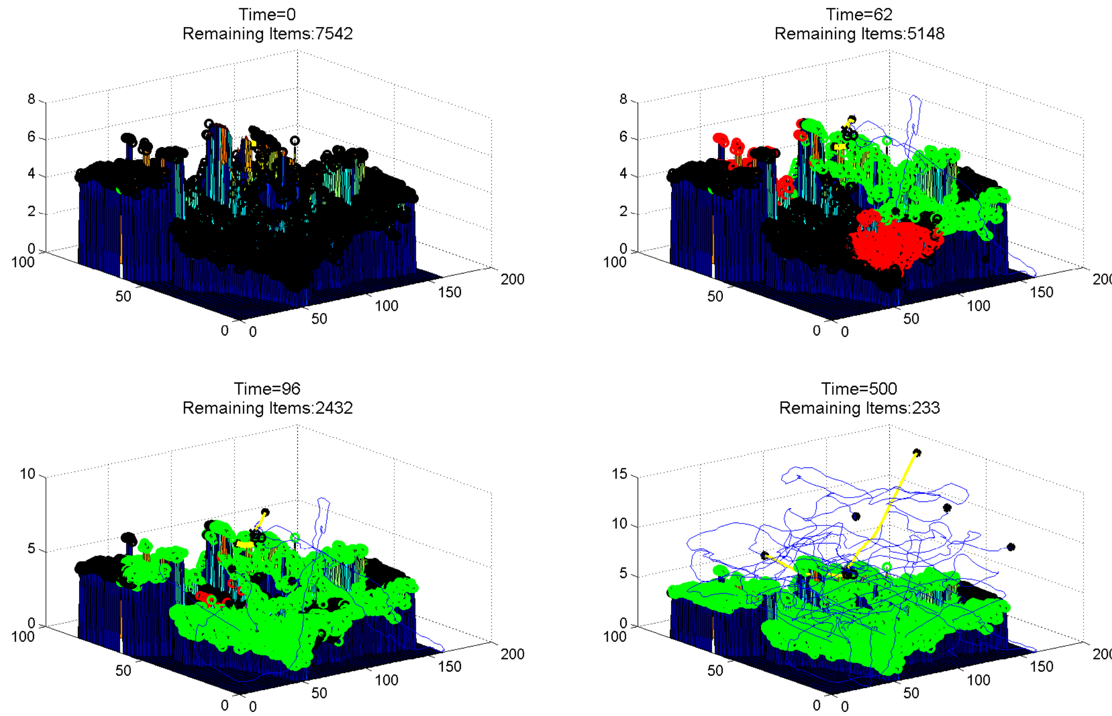

Fig. 3. 3D representation of the autonomous exploration using CAO: $N_R = 3$, $N_L = 7542$, $N_T = 2$. UAUV trajectories are illustrated using blue curves while target trajectories are depicted using yellow curves.

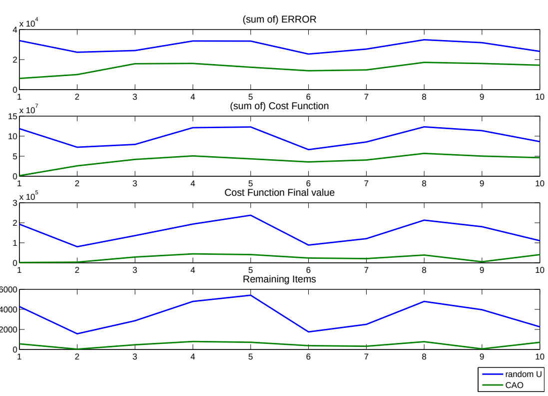

Fig. 5. Autonomous exploration using CAO and Random trajectories: $N_R = 3$, $N_L = 1000$, $N_T = 2$. Comparison of different evaluation criteria for 10 different experiments.

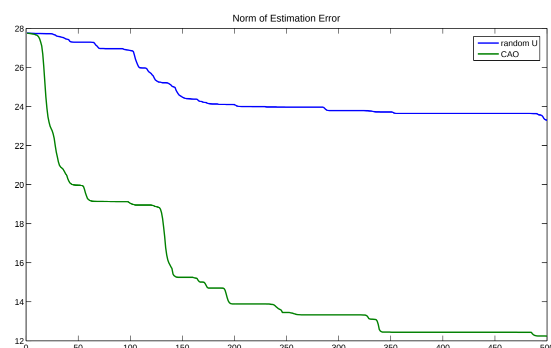

Fig. 6. Autonomous exploration using CAO and Random trajectories: $N_R = 3$, $N_L = 7542$, $N_T = 2$. Norm of Estimation Error.

VI. Conclusions

Current multi-UAUV systems are far from being capable of fully autonomously taking over real-life complex situation-awareness operations. As such operations require advanced reasoning and decision-making abilities, current designs have to heavily rely on human operators. In this paper, we presented a new approach that is capable to efficiently and fully-autonomously navigate a team of UAUVs when deployed in exploration of unknown static and dynamic environments towards providing accurate static/dynamic maps of the environment. Realistic simulation experiments exhibited the efficiency of the proposed approach.

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